Lecture 17: 10/17/2018

Bondi-Hoyle Accretion:

With the help of the three hydrodynamic equations, we can now discuss the problem of accretion onto a compact object. We start by discussing radial accretion onto an isolated compact object. In reality, compact objects tend to be surrounded by an accretion disk as matter rarely falls radially. Nevertheless, spherical accretion is the simplest way of matter accreting, and we can learn much by considering it. Moreover, there are situations that are described by radial accretion to a very good approximation such as a supermassive black hole accreting from the interstellar medium at large distances.

We consider a radial steady flow, which implies that all of the relevant quantities depend on the radial distance "r" only. The first equation of hydrodynamic (Conservation of mass) then reads:

$$\frac{1}{r^2}\frac{d}{dr}(r^2 \rho v) = 0 \Rightarrow r^2 \rho v = \text{Const.}$$

We note that the accretion rate is given by:

$$\dot{M} = 4\pi r^2 \rho v$$

Therefore, the accretion rate is constant over time. Assuming that the mass of the compact object changes very slowly due to accretion, the second equation of hydrodynamics (Conservation of momentum) becomes:

$$v\frac{dv}{dr} + \frac{1}{\rho}\frac{dP}{dr} + \frac{GM}{r^2} = 0$$

As far as the third hydrodynamic equation (Conservation of energy) is concerned, we can often get a fairly accurate solution from a polytropic relation:

$P = D\rho^{\Gamma}$ ($\Gamma = 1$; isothermal, $\Gamma = \frac{5}{3}$; adiabatic)

monatomic ideal gas

This gives us:

$$\frac{dP}{dr} = \frac{dP}{d\rho}\frac{d\rho}{dr} = c_s^2 \frac{d\rho}{dr} \qquad \left(c_s \equiv \left(\frac{dP}{d\rho}\right)^{\frac{1}{2}}; \text{ speed of sound}\right)$$

This results in:

$$v\frac{dv}{dr} + \frac{c_s^2}{\rho}\frac{d\rho}{dr} = -\frac{GM}{r^2}$$

After using the equation for the conservation of the mass, it can be rewritten as:

$$\frac{1}{2}\left(1 - \frac{c_s^2}{v^2}\right)\frac{dv^2}{dr} = -\frac{GM}{r^2}\left(1 - \frac{2c_s^2 r}{GM}\right)$$

This is the so-called "Parker wind equation" that was first derived to describe the solar wind.

Useful information can be obtained by looking at the asymptotic behavior of this equation. For accretion (which is inflow of gas), one expects that $\frac{dv^2}{dr} < 0$ for all values of r. This implies that the sign of the left-hand side of Parker equation depends on the sign of $1 - \frac{c_s^2}{v^2}$. We note that as $r \to \infty$, the right-hand side of the equation becomes positive. Hence, we must have $v < c_s$ as $r \to \infty$, which implies "subsonic" accretion. On the other hand, the right-hand side of Parker equation is negative for $r \to 0$. We must therefore have $v > c_s$ in this limit, which implies "supersonic" accretion.

Transition from subsonic to supersonic ($v = c_s$) happens at the point r_s, where:

$$1 - \frac{2c_s^2 r_s}{M} = 0 \Rightarrow r_s = \frac{GM}{2c_s^2(r_s)}$$

With this insight, we now return to the equation obtained from momentum conservation:

$$v\frac{dv}{dr} + \frac{c_s^2}{\rho}\frac{d\rho}{dr} + \frac{GM}{r^2} = 0$$

From the polytropic relation, we have $c_s^2 = D\Gamma\rho^{\Gamma-1}$ and $\frac{c_s^2}{\rho}\frac{d\rho}{dr} = \frac{D\Gamma}{\Gamma-1}\frac{d\rho^{\Gamma-1}}{dr}$ for $\Gamma \neq 1$ (for the isothermal

Case, $\Gamma = 1$, we have $c_s = $ Const. and $\frac{c_s^2}{\rho} \frac{d\rho}{dr} = c_s^2 \frac{d \ln \rho}{dr}$).

Integrating the above equation then yields:

$$\frac{1}{2} v^2 + \frac{D\Gamma}{\Gamma - 1} \rho^{\Gamma - 1} - \frac{GM}{r} = \text{Const.} \qquad (\Gamma \neq 1)$$

This is equivalent to:

$$\frac{1}{2} v^2 + \frac{c_s^2}{\Gamma - 1} - \frac{GM}{r} = \text{Const.} \qquad (\Gamma \neq 1)$$

The constant value can be found by taking the $r \to \infty$ limit. In this limit $v \to 0$, $c_s \to c_s(\infty)$. Therefore, the constant is $\frac{c_s^2(\infty)}{\Gamma - 1}$ (for $\Gamma = 1$ it will be different). As a result, we have:

$$\frac{1}{2} v^2 + \frac{c_s^2}{\Gamma - 1} - \frac{GM}{r} = \frac{c_s^2(\infty)}{\Gamma - 1}$$

At the point $r_s = \frac{GM}{2 c_s^2}$, we have $v = c_s$. Thus:

$$c_s^2(r_s) \left(\frac{1}{2} + \frac{1}{\Gamma - 1} - 2 \right) = \frac{c_s^2(\infty)}{\Gamma - 1}$$

This leads to:

$$c_s(r_s) = c_s(\infty) \left(\frac{2}{5 - 3\Gamma} \right)^{\frac{1}{2}}$$

The accretion rate is then found to be:

$$\dot{M} = 4\pi r_s^2 \rho_{(r_s)} c_s(r_s)$$

The polytropic relation implies that:

$$\rho_{(r_s)} = \rho_{(\infty)} \left[\frac{c_s(r_s)}{c_s(\infty)} \right]^{\frac{2}{\Gamma - 1}}$$

We finally arrive that the following expression:

$$\dot{M} = \pi G^2 M^2 \frac{\rho_{(\infty)}}{c^3_{s(\infty)}} \left(\frac{2}{5 - 3\Gamma} \right)^{\frac{5-3\Gamma}{2(\Gamma-1)}} \qquad (\Gamma \neq 1)$$

This is a remarkable relation that expresses the accretion rate in terms of the physical quantities at infinity (i.e., in the interstellar gas).

As an example of what this means observationally, let us consider the case of an isolated neutron star that accretes from the interstellar medium. In this case $\rho_{(\infty)} \sim 10^{-24}\ g\ cm^{-3}$ and $T \sim 10^4 K$, which results in $c_{s(\infty)} \sim 10\ km\ s^{-1}$. The accretion rate onto the neutron star is then:

$$\dot{M} \sim 1.4\times 10^{11} \left(\frac{M}{M_\odot}\right)^2 \left[\frac{\rho(\infty)}{10^{-24}\,g\,cm^{-3}}\right] \left[\frac{c_s(\infty)}{10\,km\,s^{-1}}\right]^{-3} g\,s^{-1}$$

The accretion luminosity is found to be:

$$L_{acc} = \frac{G M \dot{M}}{R_{NS}} \sim 2\times 10^{31}\ erg\,s^{-1}$$

This is detectable out to a distance of ~1kpc with the current instruments.